Discussion of:
"Nonparametric Stochastic Discount Factor Decomposition"
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Testing Asset Pricing Models (APMs)

- Hansen and Jagannathan (1991) use data on assets returns to impose restrictions on APMs.

- The literature proposed several extensions of the HJ variance bounds including entropy, higher moments, and more general convex functions of the SDF.

- A more recent approach looks at long-term implications of APMs as Alvarez and Jermann (AJ, 2005), Hansen and Scheinkman (HS, 2009),

Testing APMs - Long Term Implications

- AJ(2005), HS(2009), Hansen(2012), and Borovicka et. al (2012) decompose the SDF into a permanent and a transitory term.

- Results: APMs have to be able to generate high variability on their permanent SDF component.

- Therefore long-term approach is useful to better distinguish between APMs

- This paper: Nonparametric methodology to decompose the SDF into permanent and transitory terms.
Stochastic Discount Factor Decomposition and Long Run Risks

- Given a certain SDF $M_{t,\tau}$, HS (2009) show that its permanent component can be written as:

$$\tilde{M}_{t,\tau} = \exp(-\rho\tau)M_{t,\tau} \frac{\phi(X_{t+\tau})}{\phi(X_t)} \quad (1)$$

- where $\phi(.)$ is a principal eigenfunction of the linear operator implied by the SDF $M$, solving:

$$E_t(M_{t,\tau}\phi(X_{t+\tau})) = \exp(\rho\tau)\phi(X_t) \quad (2)$$

- $\tilde{M}$ is a martingale and therefore shocks have a permanent effect.
Main Contribution

- Given an SDF, Christensen (2012) proposes a method to approximate the principal (eigenfunction, eigenvalue) pair to obtain the permanent component (martingale).

- **Simple:** Extracts the martingale by solving a matrix eigenvector problem based on observed data and given SDF.

- **Nonparametric:** To extract the martingale needs the SDF but no specification of the dynamics of the state vector on the economy.

- **Interesting features:**
  - Robust to misspecification on the dynamics of observable variables (consumption) on the state vector. Martin (2012)
  - Robust to the existence of Habit formation.

- But, what kind of APM can be tested with this methodology?
Limitations: Long Run Risk Models

- The classic CCAPM can be tested easily since the state vector includes only consumption growth $X_t = G_t$ which is observable.

- However can not be applied to any model whose implied SDF is a function of non-observable state variables.

- That includes the whole Long Run Risk literature started by Bansal and Yaron (2004).

  - Dynamics of the state vector is necessary since the SDF depends on a return to a claim on consumption.

  - State vector includes conditional expected consumption growth, and consumption growth time-varying volatility, both non-observable.

- Can only apply if specify the dynamics... Still interesting as an econometric method to extract the martingale, assumed a known dynamics.
A Suggestion on the Focus

- Paper is motivated based on the idea that for robustness purposes it is important not to specify the state vector dynamics.

- True but has limitations as shown.

- An alternative use for your method is to extract the martingale part of the SDF from APMs with specified more complex dynamics.

- In this context, a comparison with what has been done by Hansen (2012) and Borovicka et. al (2012) would be useful.

  - Would you obtain similar sensitivities/results with your method when adopting their specified dynamics?
  - Would small changes in the structure of the dynamics generate very different results?
Given an SDF, Backus, Chernov and Zin (2012) suggest calculating one period entropy and conditional entropies for different horizons.

The limit of their conditional entropies when horizon increases converges to the entropy of the permanent SDF component.

In principle, there is no need to specify the dynamics of the state vector as long as the SDF is known.

When testing APMs, is there any advantage in adopting the proposed method as opposed to BCZ? Maybe knowledge of the eigenfunction?
Limitations 2: Recursive Utilities

- To apply the methodology, need to solve for the continuation value of the utility to have an implied SDF.

- Borovicka et. al (2012) solve the SDF for a specific affine dynamics of the state vector. (dynamics is needed!)

- Hansen (2012) and Hansen and Scheinkman (2012) approximate the SDF with a trick by solving an eingefunction problem with power utility.
  - Less interesting case: SDF similar to that of a habit formation model.
  - The recursive structure contributes only to the transitory component.

- Any thoughts on how to solve this problem without specifying the dynamics?
Conclusion

- Proposes a simple nonparametric method to extract the permanent component of an SDF implied by an APM.

- Allows for analysis of model long-term properties (related to a large recent literature).

- Chosen a basis of functions (sieves), the estimator solves for a finite dimensional eigenvector problem based on data and the SDF.

- Nice asymptotic properties of estimators of eigenfunction, eigenvalue, and long-term measures.

- Limitation: Not useful for LRR models, and other recursive utility models, unless dynamics of state variables are specified.

- Question: Is your estimator any different from the sieves method proposed at Section 7 of Chen et al. (2009)? Better formalized?