Discussion of:
"Tail Risk Premia and Return Predictability"
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Tail Risk, Tail Risk Premia and Predictability

- Variance is a priced risk factor.
  - Variance Risk Premium (VRP) predicts market future returns (Bollerslev et al. 2009).

- The VRP can be decomposed into diffusive, small-jumps and large-jumps risk.

- This paper: Extracts the large-jump tail risk premium from S&P 500 market based on BT (2013).
  - Short-maturity OTM options to estimate two time-varying parameters for the risk-neutral Levy measure for jumps.

- The jump tail premium is an important component of VRP.
  - Most of VRP’s forecasting power due to jump premium.

- It significantly forecasts future market, size, book to market, momentum and industry portfolio returns.
A closer look at this paper

- Given the stochastic evolution of the price $X$ of a risky asset:
  \[
  \frac{dX_t}{X_t} = a_t dt + \sigma_t dW_t + \int_\mathbb{R} (e^x - 1)(\mu(ds, dx) - dt\nu_t^P(dx)) \quad (1)
  \]

- And its return quadratic variation over an interval $[t, t + \tau]$
  \[
  QV_{[t, t+\tau]} = \int_t^{t+\tau} \sigma_s^2 ds + \int_t^{t+\tau} \int_\mathbb{R} x^2 \mu(ds, dx) \quad (2)
  \]

- Absence of arbitrage guarantees the existence of at least one risk-neutral measure $Q$ under which:
  \[
  \frac{dX_t}{X_t} = (r_t - \delta_t) dt + \sigma_t dW_t^Q + \int_\mathbb{R} (e^x - 1)(\mu(ds, dx) - dt\nu_t^Q(dx)) \quad (3)
  \]

- The VRP is the difference between the expected quadratic variation under the objective and risk-neutral measures:
  \[
  VRP_{t,\tau} = \frac{1}{\tau} (E_t^P (QV_{[t, t+\tau]}) - E_t^Q (QV_{[t, t+\tau]})) \quad (4)
  \]
A closer look at this paper: continues...

- The VRP can be decomposed into diffusive and jumps risk, and for short time-intervals (τ close to t) only the jump premium matters.

\[
\lim_{\tau \downarrow t} VRP_{t,\tau} = \int_{\mathbb{R}} x^2 (\nu_P^t(dx) - \nu_Q^t(dx)) + \text{variance jump premium} \quad (5)
\]

- This paper assumes that there is no variance jump premium.

- Interested in tail jump premium:

\[
LJV^Q_{[t,t+\tau]} = \int_{t}^{t+\tau} \int_{x <- k_t} x^2 \nu_Q^s(dx)ds \quad (6)
\]

- Now, further assuming that P-jump tails are orders of magnitude smaller than Q-jump tails:

\[
LJP_{t,\tau} - RJP_{t,\tau} \approx -\frac{1}{\tau} E_t^Q \left( \int_{t}^{t+\tau} \int_{x <- k_t} x^2 \nu_Q^s(dx)ds \right) \quad (7)
\]

- Obtains a model-free proxy for investors fear index.
A closer look at this paper: continues...

- Semi-parametric identification of the jump-premium via $\nu_t^Q$.

- More general than...
  - parametric studies like Pan (2002), Broadie et al. (2007),
    - semi-parametric identification of Todorov (2010)

- New important feature: Probabilities of having new jumps are time-varying and not time-homogeneous across different jump sizes.

- Show that new feature improves forecasts of future market returns and other equity returns.

- Question: Why to eliminate variance jump premium? Harder to identify but potential to increase predictability.
An Asset Pricing consistency perspective

- Results based on the existence of a risk-neutral probability $Q$ under which tail risk premia is extracted.

- In any pricing model, factors that drive risk premia are precisely the factors that drive the variance of the SDF.

- Therefore, tail risk premia extracted from S&P 500 data should be a priced factor in the cross section of stocks.

- The returns of stocks within this economy should satisfy:

$$E(R_i) = R_f + \beta \lambda + \beta_{TR} LJV + \epsilon$$

- Running a cross-section regression including Fama and French factors, $VRP - LJV$, and $LJV$ as risk factors:

- The exposure to $LJV$ ($\beta_{TR}$) should appear significant and contribute to a better explanation of expected returns.
Misspecification of the nonparametric model

- Dependence of log-price option pricing errors across different strikes might generate biased results for $\alpha_t$ and $\phi_t$.

- Hard to imagine that there won’t be strong error dependence across (close) strikes.
  - Two stage estimator where first estimate $\alpha_t$ and use $\hat{\alpha}$ to estimate $\phi_t$ should be affected by cross-sectional error dependence.

- Tests for such possibility of bias?

- Could estimate Pan’s (2002) or Andersen, Fusari and Todorov (2013) model with available option panel to obtain option pricing errors.

- Test for dependence on these errors.

- Design Monte Carlo experiment or bootstrap errors to obtain distribution of the estimators.
Multiple Risk-Neutral Measures (RNMs) and identification assumptions

- Market is incomplete since jumps can’t be hedged: Infinity of RNMs. Which one to choose?

- Bollerslev and Todorov (2013) uniquely identify RNM using cross-section of OTM options.

- Semi-parametric jump intensity process:

  \[ \nu_t^Q(dx) = \phi_t^+ e^{\alpha_t^+ x} 1_{\{x > 0\}} + \phi_t^- e^{-\alpha_t^- |x|} 1_{\{x < 0\}} dx, |x| > k_t \]

- From Girsanov’s theor.: \( \nu_t^Q(dx) = Y(X)\nu_t^P(dx) \), with \( Y \) strictly positive charact. jump premium.

- This paper free of \( P \)-dynamics: magnitude of risk-neutral tails much bigger than objective ones.

- If necessary to model \( P \), similar structure for intensities \( \nu_t^Q,\nu_t^P \) restricts risk premia.

- How to generalize Bollerslev and Todorov (2011)?
Can we obtain return predictability with alternative strategies involving options?

- Vix reflects the floating leg of a variance swap under certain restrictions for the S&P 500 dynamics.

- Martin (2011) proposes a simple variance swap and corresp. index SVIX, robust to jumps in S&P dynamics.

- Moreover, SVIX represents model-free lower bound for the equity premium.

  - The difference between VIX and SVIX reflects a trade in skew highly correlated with fear index of BT (2011).

- Suggest comparing predictability results of LJV with those from SVIX, and skew swaps.

- Comparison will indicate the importance of using their methodology as opposed to traded strategies.
Predictability, Option Data and Tail Jump Premia

- Options contain strong information about volatility and jump risk premia (Bates, 2000 Pan, 2002).

- In especial OTM options contain important information about tails behavior.

- Question: How much of the predictability comes from the new nonparametric techniques for tail estimation when compared to other alternatives that use OTM options?


2. Use short-dated option returns to estimate nonparametric RND and calculate variance of observations that exceed a threshold to represent tail jump premium.

Compare predictability obtained with theirs.
On the increasing predictability of LJV with horizon

- Jumps have a short-horizon effect in the way they affect the volatility of prices (fast mean reversion).

- Intuitively we could expect jumps to affect short horizon returns.

- However, compensation for jumps (LJV) has better return predictability for longer horizons.

- How to reconcile long-horizon predictability of LJV with fast mean reversion of volatility caused by jumps?

- Jump premium should be a persistent process.

- This persistence can be estimated using the parametric structure of LJV: \( LJV_t = g(\phi_t, \alpha_t) \).

- Or fitting an AR model to LJV. AR coefficient expected to be high.
Robustness Predictability Tests for Jump Premia

- Following Goyal and Welch (2008) testing for out-of-sample predictability should give stronger support for the tail risk premia measure as a predictor.

- Also, including other variables that can potentially behave similarly to LJV on multi-variate regressions: VIX, SVIX, VIX-SVIX