Discussion of:
”Hedging in Fixed Income Markets”
by
A. Malkhozov, P. Mueller, A. Vedolin and G. Venter

Caio Almeida
Getulio Vargas Foundation

Linking the Mortgage and Bond Markets

- Simple equilibrium term structure model that links the mortgage and bond markets.

- Key ingredient comes from financial intermediaries optimization problem: Market prices of bond risks are a linear function of mortgage duration.

- Theoretical model provides important testable implications:
  - Bonds yields, and bond excess returns should be explained by mortgage duration, and bond volatility by convexity.

- Nice empirical results.
A No-Arbitrage Perspective

- Given a probability space \((\Omega, F, P)\), we have a one factor Vasicek (1977) model augmented with a Duration factor.

\[
dr_t = \kappa (\theta - r_t) dt + \sigma dB_t
\]  
\[
dD_t = -\kappa D_t dt + \eta_Y \sigma \frac{B(\bar{\tau})}{(1 - \eta_Y C(\bar{\tau}))} dB_t
\]

- Only one Brownian Motion drives both factors.

- Absence of arbitrage implies the existence of an equivalent risk-neutral measure \(Q\) under which bond prices are given by:

\[
P_t^{\tau} = E^Q \left[ e^{-\int_t^{t+\tau} r_u du} \right]
\]
No-Arbitr. Perspective: Why Do Yields Depend on $D_t$?

- Model Result: Bond yields are affine on the two factors:
  \[ y_t^\tau = A(\tau) + B(\tau)r_t + C(\tau)D_t \]  
  (4)

- Why do yields $y_t^\tau$ depend on $D_t$ if the short-rate $r_t$ and its dynamics are not functions of $D_t$?

- From a no-arbitrage perspective, two-factor affine model with restricted market prices of risk ($\lambda_t = -\alpha \sigma_y D_t$).

- Applying Girsanov’s theorem note that the risk-neutral short-rate dynamics depends on $D_t$:
  \[ dr_t = [\kappa(\theta - r_t) - \sigma \alpha \sigma_y D_t]dt + \sigma dB_t^Q \]  
  (5)

- Distinct from usual no-arbitrage affine models where the short-term rate is linear in all risk factors.
Two model ingredients generate two additional testable empirical implications (not verified in the paper).

1. Innovations of the duration factor $D_t$ and the short-rate $r_t$ are the same.

2. Mortgage duration $D_t$ and short-rate $r_t$ are the only factors driving yields dynamics and spanning the cross section of yields.

   Therefore, duration $D_t$ should be spanned by bond yields $y_t^\tau$. 
Empirically Testable Points

1. Innovations of duration factor and short-rate should be highly correlated.
   - Suggest running a Svensson model on bond yields to estimate the short-term rate.
   - Estimate AR(1) processes for $r_t$ and $D_t$.
   - Estimate correlation between innovations.
   - In the data, correlation bet. innovations of $D_t$ and $y_{0.5}$ is 26.7%.

2. The cross-section of yields should span the duration factor.
   - In the paper: Regression of duration on 5 PCs, but do not report the $R^2$.
   - Model suggests a high $R^2$, while I found $R^2 = 0.35$.
   - Existence of information on the mortgage market not capture by bond yields. Unspanned duration?
Model Reconciliation with Term Structure Movements

- Usually three sources of risk (PCs) necessary to capture variability on yields (Litterman and Scheinkman, 1991).

- Model presents two factors with one unique source of risk: Incapable of endogenously capturing yields dynamics.

- Suggestion: Slightly more general dynamics for $D_t$ with extra independent source of (Brownian) risk.

\[ dD_t = -\kappa_D D_t dt + \eta_y (dy_t^\tau - E_t(dy_t^\tau)) + dB_t^2 \]  \hspace{1cm} (6)

- Keeps economic motivation: Duration dynamics still depends on innov. of long-term yield.

- Easy to solve: Model keeps the same structure but now...

- Two-factor affine Gaussian model with restricted market prices of risk and two sources of risk.
Unspanned Volatility versus Unspanned Duration

- Duration is not spanned by the short-term rate but enters on its risk-neutral dynamics and is spanned by bond yields.

- Similarly in USV models (Collin-Dufresne and Goldstein, 2002) volatility is not spanned by the short-term rate and by bond yields but enters in the short-rate dynamics.

- Although the two models have apparently a similar structure the model in this paper generates a complete bond market while USV generates an incomplete market.
In the model, the risk-neutral mean-reversion rate of $D_t$ is\[
\kappa^O_D = \kappa_D - \alpha \eta_y (\sigma_y^\tau)^2\]
where $\alpha$ is the RA coefficient.

If we have to restrict $\kappa^O_D$ to be positive this imposes an additional limit to the risk-aversion coefficient $\alpha$.

Since $\sigma_y^\tau$ is endogenous, it would be interesting to see if this restriction is binding for reasonable sets of parameters.