Discussion of:

"Pricing Default Events: Surprise, Exogeneity and Contagion"

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Relaxing the No-Jump Condition on Defaultable Models

- Large literature price defaultable claims with DTSM machinery (Duffie and Singleton, 1999).

- Limitation, no-jump condition: Asset value can not jump at default time.

- Excludes important existing cases:
  1. Systematic Jump Risk: Default is a priced risk appearing in the SDF.
  2. Counterparty credit risk: Current defaults affect conditional probabilities of future defaults.
     - Example: Intensity of counting process changes with occurrence of defaults.
     - Breaks doubly stochastic condition.
  3. Flight to quality: Risk-free rate goes down at a default.
This Paper: Multiple Contributions

- Explores, on a discrete setting, the effects of allowing the SDF to be an exponential-affine function of factors and counting processes driving defaults.

- Implications on the causality relation between factors $F_t$ and default processes $d_t$ under the risk-neutral measure.

- Theoretical tractable framework that accommodates the three above-mentioned violations of the no-jump condition.

- From a pricing perspective, closed-form formulas for defaultable bonds and other credit derivatives.
Contribution with Respect to the Literature

- **Interest rates**: Pricing interest-rate instruments (HJM, 1992),
  - HJM-like models: Start with risk-neutral measure and given the current term structure, price derivatives.

- Analyzing risk-premium properties (Duffee(2002), Dai and Singleton (2002)),
  - DTSMs estimate factors’ risk-premium using both the physical ($P$) and risk-neutral ($Q$) measures (panel data).

- **Credit risk**: Many papers: Risk-neutral measure and existence of intensities for the counting processes driving default.

- This paper: Framework useful for pricing ($Q$), and analyzing the risk-premium of default (look at $P$ & $Q$).
Two related results in this paper are:

1. An intensity can exist under the historical world without existing in the risk-neutral world. (prop. 4)

2. Assuming the existence of an intensity in the risk-neutral world we implicitly do not price the surprise events. (prop. 5)

1. and 2. have important implications since most papers assume the existence of an intensity model under both the physical and risk-neutral measures.

Different definitions for the intensity play a role on results.

This paper adopts definition from Duffie and Garleanu (2001), which relies on doubly stochasticity: $\lambda_{t+1}$ is $\Omega^*_t$-measurable.

In this case, causality of default process $d_t$ on factors $F_t$ implies inexistence of an intensity.
Is There an Intensity Under $Q$ when Default is Priced?

- Alternative definition (Duffie, 2001): $\lambda$ is a nonnegative predictable process with $M_t = N_t - \int_0^t \lambda_u du$, $\forall t$ defining a local-martingale.

- Others (Yu (2007), CDGH (2004)) define it as adapted to a right-continuous filtration that includes events generated by $N_t$.

- Girsanov’s theorem for counting processes:
  \[
  \xi_t = \exp\left(\int_0^t (1 - \eta_s)\lambda_s ds\right) \prod_{\{i: T(i) \leq t\}} \eta T(i)
  \]
  is a local-martingale

- Bounded $\eta$ and $\lambda$ guarantee existence of an equivalent measure $\tilde{Q}$ where $\eta \lambda$ is the $\tilde{Q}$-intensity of $N_t$.

- Taking the SDF to be $M_t = e^{-\int_0^t r_u du} \mathbb{E}(\text{brownian m.p.r}) \xi_t$

- Get priced defaults and existence of an intensity under the risk-neutral measure $Q$. There are multiple EMMs.
Still on Propositions 4 and 5

- The previous example shows that modifying the measurability condition in the definition of intensity eliminates the problem raised in proposition 4:
  - We can have priced default events and existence of an intensity under the risk-neutral measure.

- Contradicting proposition 5: Take $\lambda$ constant and $\eta$ deterministic and we will have a bond priced with $B(t, h) = E_t^Q[\exp(-\int_t^T (r_u + \lambda \eta(u))du)]$ with risk-neutral intensity $\lambda \eta$ and default risk-premia equal to $\eta$.

- Importance of proposition 5: If our purpose is only pricing, similarly to the HJM case, shouldn’t we forget about risk-premia and start directly under a risk-neutral measure with an intensity process according to realistic patterns?
Related Literature on Pricing Contagion Risk and Systematic Credit Risk

- Yu (2007) obtains joint distributions of default times of individual firms when there are multiple firms with counterparty credit risk

- Collin Dufresne, Goldstein, Hugonier (CDGH, 2004) suggest a change of probability measure giving zero probability to paths where default occurred prior to maturity of the credit derivative.
  - Recover the doubly stochastic pricing formula, under a different probability measure.
  - CDGH analyze cases of systematic jumps in the SDF, counterparty risk, and flight to quality.

- This paper provides the most tractable framework for empirical analysis of credit markets.