Discussion of:

"Estimation of Dynamic Asset Pricing Models with Robust Preferences"

by

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In asset pricing, GMM is usually adopted to estimate SDFs with observable components, with law of motion of stochastic components not fully specified:

\[ E_t(m_{t+1}(\theta)R_{t+1}) = 1 \]

With that approach, there is no room for estimation of SDFs coming from economic models with recursive utility.

Recently Chen, Favilukis and Ludvigson (QE, 2013) attacked this problem providing a solution.

This paper: Generalizes CFL (2013) to consider robust preferences (capturing in particular recursive util.), but also allowing departure from Rational Expectations.

Important contribution to the literature of asset pricing and macro-finance.
Idea is very exciting but results are still preliminary in at least two dimensions...

1. Needs a more clear position in terms of comparison with Chen, Favilukis and Ludvigson (2013).
   - Very particular case of recursive utility considered, when $EIS = 1$. Known to be a problematic case.

2. Allowing for departures from R.E. makes it hard to disentangle differences in beliefs from risk-aversion. (Hansen, JPE, 2014)
   - It would be interesting to control the size of departure from R.E.
Recursive Utility with $EIS = 1$

- When $Q = P$ robust preferences reduce to standard Epstein-Zin utility with $EIS = 1$.

- In such case, implied SDF (even with higher RA) does not generate enough variability to pass the entropy bounds.

- However, in similar settings like in Bansal and Yaron (JF, 2004) the implied SDF appears to have large variability even with small RA. But $EIS > 1$

- My guess is that $EIS = 1$ shuts down a big part of the power of recursive utility.

- Confirmed by Chen, Favilukis and Ludvigson (2013): $EIS = 1 + \log\text{-linear consumption}$ imply analytical continuation value, which is function of observable consumption (not interesting for your method).

- Nice to understand contributions to robustness formulations that are not a particular case of recursive util.
Departure from R.E

- In the empirical section, important action (SDF variability) comes from the change of measure that measures departures from R.E.

- But, is its magnitude reasonable, if expected to be a small departure from R.E.?

- Hansen (2014) suggests ways to discipline the departure from R.E models: Large deviations, bounds based on the Cressie Read family.

- Change of measure is chosen to minimize pricing errors in the conditional Euler equations. Unique given choice of exponential function representing it.

- Is it too big?

- I will suggest an alternative way of estimating the change of measure using GEL estimators.
Robustness, Recursive Preferences, and Misspecific.

- Those three ingredients appear in the paper but their separate roles are not clear. Needs clarification.

- Some questions that might help:
  - 1. Can we always represent recursive utility using robust preferences under R.E? No, recursive util. more general.
  - 2. Is there misspecification without recursive preferences? Yes. See example later.

- 1. suggests exploring in more details the relationship between robustness and recursive utilities.

- Is it possible that different metrics from entropy in the continuation recursivity equation for the robust agent generate more general recursive preferences? Apparently this was not explored in the literature.

- 2. helps to disentangle. As a byproduct, provides alternat. way to measure magnitude of departure from R.E.
Disentangling Robustness from Recursive Prefer.

- Start with model whose implied SDF $S(\theta, V_{t+1})$ depends on parameters $\theta$ and a continuation value coming from an Epstein-Zin type utility.

- Disentangle misspecification (via robustness) from recursive utilities.

- Robustness will only appear via a departure from R.E. model with no recursivity on it!

- Agent considering misspecification choosing from a set of probability measures that help the model to satisfy the Euler equations implied in equilibrium:

$$\hat{M}_{t+1}^{KLIC}(\theta) = \arg \min_{M_{t+1}, V_{t+1}} E_t[M_{t+1} \log(M_{t+1})]$$

s.t. $E_t(M_{t+1} \ast S(\theta, V_{t+1} R_{t+1})) = 1, E_t(M_{t+1}) = 1.$
Disentangling Robustness from Recursive Preferences, continues...

- The minimum entropy measure

\[ \hat{M}_{t+1}^{KLIC}(\theta) = \frac{\exp(-\lambda S(\theta, \hat{V}_{t+1}) R_{t+1})}{E_t[\exp(-\lambda S(\theta, \hat{V}_{t+1}) R_{t+1})]} \]

- is used to distort model solution. Now, choose \( \theta \) to minimize \( E_t[M_{t+1}(\theta) \log(M_{t+1}(\theta))] \).

- Different interpretation from framework of Hansen and Sargent (2008):
  - Chooses measure that corrects model in the pricing equation, and which has smallest entropy versus, in the paper, measure that provides worst utility for that agent and that has smallest entropy.

- Approach from literature on model misspecification with GEL estimators (Almeida and Garcia, JoE, 2012).
Minimum Discrepancy Approximation of Proxies: Multiplicative Case

- Given a proxy asset pricing model $S$, and a convex discrepancy function $\phi$, find an admissible SDF which is as close as possible to $S$ in the $\phi$ discrepancy sense:

$$ \tilde{\delta}_{MD}^{mu}(\theta) = \min_{m \in L^2} E\{\phi(m)\} \text{ subject to } E(mS(\theta)R) = 1 $$ (1)

- The above problem can be transformed to the following more convenient one:

$$ \tilde{\delta}_{MD}^{mu}(\theta) = \min_{\tilde{m} \in L^2} E \left\{ \phi \left( \frac{\tilde{m}}{S(\theta)} \right) \right\} \text{ subject to } E(\tilde{m}R) = 1 $$ (2)
Multiplicative Case under Cressie Read Discrepancies

Let $\phi$ belong to the Cressie Read family: $\phi(\pi) = \frac{\pi^{\gamma+1} - 1}{\gamma(\gamma+1)}$. For a fixed $\theta$, the optim. problem (2) specializes to:

$$\tilde{\delta}_{CR}^{mu}(\theta) = \min_{m \in L^2} \mathbb{E} \left\{ \frac{(\tilde{m}/S(\theta))^{\gamma+1} - 1}{\gamma(\gamma + 1)} \right\} \text{ subject to } \mathbb{E}(\tilde{m}R) = 1 \quad (3)$$

Then the dual GEL problem to the MD problem is given by:

$$\tilde{v}_{CR}^{mu}(\theta) = \max_{\tilde{\lambda} \in \mathbb{R}^n} \tilde{\lambda}'q - \mathbb{E} \left\{ \frac{(1 + \gamma S\tilde{\lambda}' \cdot R)^{\gamma+1}}{\gamma + 1} \right\} \quad (4)$$

and the MD multiplicative admissible SDF is given by:

$$\tilde{m}_{CR}^{mu}(\theta) = S(1 + \gamma S\tilde{\lambda}'_{mu} \cdot R)^{\frac{1}{\gamma}} \quad (5)$$
Empirical Section: Bounds to test Implied SDFs

- In the paper tests of entropy bounds are provided for the implied SDFs and for permanent components of implied SDFs.

- Almeida and Garcia (2014) have shown that SDFs implied from recursive utility cases...

- ...like the LRR of Bansal and Yaron (JF, 2004) and Valuation Risk of Rebelo, Albuquerque and Eichenbaum (2014...)

- ...since not very much skewed, have more difficulties in passing the KLIC entropic bound (Stutzer, JoE, 1995) and HJ bound with positivity constraint.

- It would be interesting to consider such bounds in addition to the entropy bounds of Bansal and Lehman (MaD, 1997).